1060 – Day 19

Fall 2013

Verify The Identity

$$\frac{\csc(x)}{2\cdot\cos(x)} = \csc(2x)$$

$$\frac{\csc(x)}{2\cdot\cos(x)} = \frac{\frac{1}{\sin(x)}}{2\cdot\cos(x)}$$ Definition of csc(x)

$$= \frac{\frac{1}{\sin(x)}}{2\cdot\cos(x)}\cdot\frac{\sin(x)}{\sin(x)}$$ multiply by $\frac{\sin(x)}{\sin(x)}$ to simplify the top

$$= \frac{1}{2\cdot\cos x\cdot\sin x}$$

$$= \frac{1}{2\cdot\sin x\cdot\cos x}$$

Remember
$\sin(2x) = 2\cdot\sin x\cdot\cos x$

$$= \frac{1}{\sin(2x)}$$ by double ∡ identity for sin(2x)

$$= \csc(2x)$$ by definition of csc(x)

(skip in class)

Eg: Verify the identity

$$\tan^2(x) + 4 = \sec^2(x) + 3$$

$$\tan^2(x) + 4 = \frac{\sin^2(x)}{\cos^2(x)} + 4$$ defn of ~~[illegible]~~ tan(x)

$$= \frac{\sin^2(x)}{\cos^2(x)} + \frac{4}{1} \cdot \frac{\cos^2(x)}{\cos^2(x)}$$ obtain like-denominators

$$= \frac{\sin^2(x) + 4\cos^2(x)}{\cos^2 x}$$ (algebra)

Remember: $\sin^2(x) + \cos^2(x) = 1$

$$= \frac{\sin^2(x) + \cos^2(x) + 3\cos^2(x)}{\cos^2(x)}$$ (algebra)

$$= \frac{1 + 3 \cdot \cos^2(x)}{\cos^2(x)}$$ pythagorean identity

$$= \frac{1}{\cos^2(x)} + \frac{3\cos^2(x)}{\cos^2(x)}$$ (algebra)

$$= \sec^2(x) + 3$$ Definition of sec(x)

Solving a trig equation

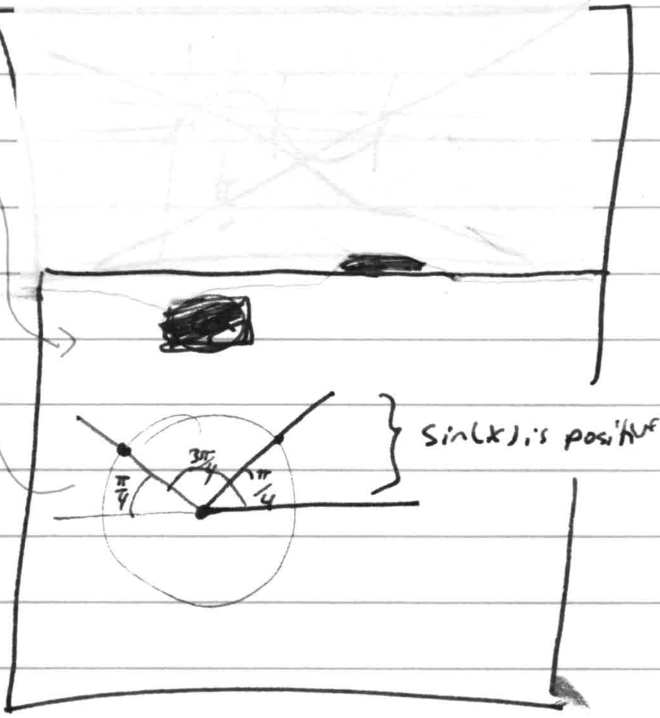
Eg: Find the values of x in $[0, 2\pi]$ such that

$$\sin(x) = \frac{\sqrt{2}}{2}$$

happens when $x = \frac{\pi}{4}$

and when $x = \frac{3\pi}{4}$

in $[0, 2\pi]$



Eg: find the x in $[0, 2\pi]$ such that

$$2 \cdot \sin x - 1 = 0$$

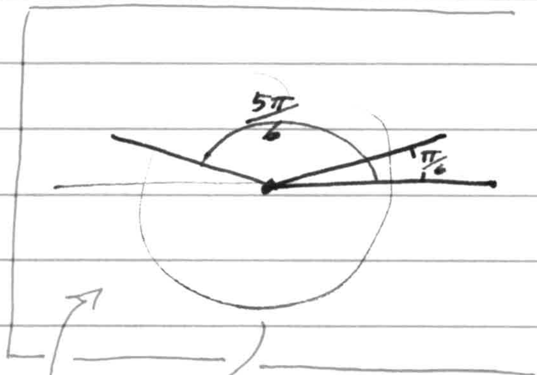
Need to rewrite this!

$$2 \cdot \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$\Leftrightarrow$$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$ in $[0, 2\pi]$



Eg: solve for x in $[0, 2\pi]$

$$\sin(x) + \sqrt{2} = -\sin x$$

$+\sin(x)$ $\qquad$ $+\sin(x)$

Rewrite first.

collect like terms

$$2\cdot\sin(x) + \sqrt{2} = 0$$

$-\sqrt{2}$ $\qquad$ $-\sqrt{2}$

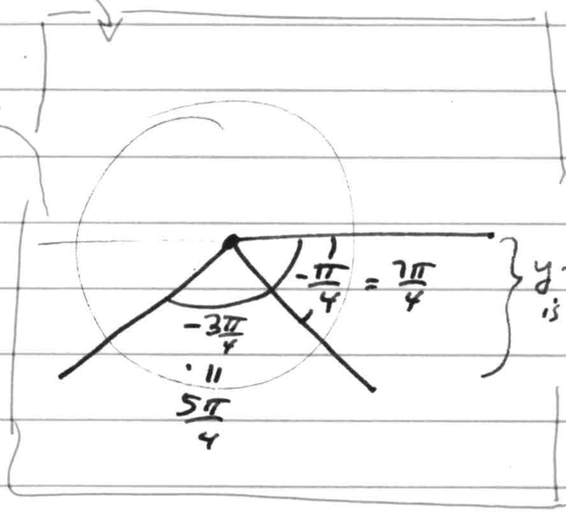
$$2\cdot\sin(x) = -\sqrt{2}$$

$$\sin(x) = \frac{-\sqrt{2}}{2}$$

$$\Leftrightarrow$$

$$x = \frac{7\pi}{4} \text{ or } \frac{5\pi}{4}$$

in $[0, 2\pi]$



y-coo
is neg

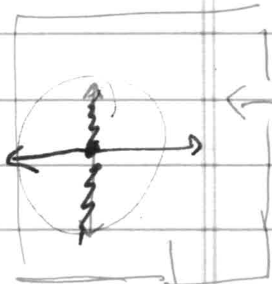
Eg: solve for x in $[0, 2\pi]$

$$\sin(x) \cdot \cos(x) = \sin(x)$$

$$\sin(x) \cdot \cos(x) - \sin(x) = 0$$

$$\sin(x) \cdot \Big(\cos(x) - 1 \Big) = 0$$

$$\Longleftrightarrow$$



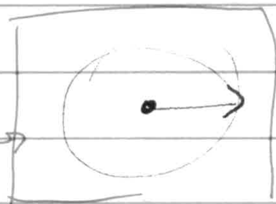
$\sin(x) = 0$ or $\cos(x) - 1 = 0$

$\Longleftrightarrow$ $\quad\quad\quad\quad$ $\cos(x) = 1$

$x =$ [redacted] 0 $\quad\quad\quad\quad$ $\Longleftrightarrow$

or $\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $x = 0$

$x =$ [redacted] π



So the equation holds for x in $[0, 2\pi]$

$\Longleftrightarrow$

$x = 0$ or π [redacted].

Eg: Solve for x in $[0, 2\pi]$

$$2\sin^2(x) = \sin(x) + 1$$

(Rewrite to get 0 on one side)

$$2\cdot(\sin(x))^2 - \sin(x) - 1 = 0$$

Think:

$$2a^2 - a - 1 = 0$$

$$\Longleftrightarrow$$

$$(2a + 1)(a - 1) = 0$$

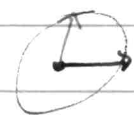
$$(2\cdot\sin(x) + 1)(\sin(x) - 1) = 0$$

$$\Longleftrightarrow$$

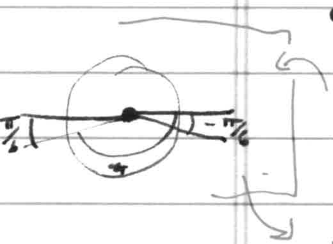
$2\cdot\sin x + 1 = 0$ OR $\sin(x) - 1 = 0$

$2\cdot\sin x = -1$ $\qquad$ $\sin(x) = 1$

$\qquad\qquad\qquad\qquad\Longleftrightarrow$



$\sin(x) = -\frac{1}{2}$ $\qquad$ $x = \frac{\pi}{2}$



$$\Longleftrightarrow$$

$$x = \frac{11\pi}{6} \text{ or } \frac{7\pi}{6}$$

so the equation holds

$$\Longleftrightarrow$$

$$x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \text{or } \frac{11\pi}{6}$$

Eg: Solve for x in $[0, 2\pi]$

$$2\cdot(\sin(x))^2 + 3\cdot\cos x - 3 = 0$$

NEED: one trig fuction

use pythagorean identity

$$(\sin x)^2 + (\cos x)^2 = 1$$

$$\Rightarrow (\sin x)^2 = 1 - (\cos x)^2$$

$$2\left(1-(\cos(x))^2\right) + 3\cdot\cos(x) - 3 = 0$$

$$2 - 2\cos^2(x) + 3\cdot\cos(x) - 3 = 0$$

$$-2\cdot\cos^2(x) + 3\cdot\cos(x) - 1 = 0$$

(mult both sides by -1)

$$2\cdot\cos^2(x) - 3\cdot\cos(x) + 1 = 0$$

think: $2a^2 - 3a + 1 = 0$

$$\Leftrightarrow$$

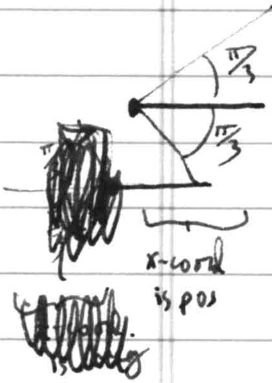
$$(2a-1)(a-1) = 0$$

$$(2\cos(x) - 1)(\cos(x) - 1) = 0$$

$$\Leftrightarrow$$

$\cos(x) = \frac{1}{2}$ or $\cos(x) = 1$

$$\boxed{x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \text{ or } x = 0}$$



you can also use
other trig identities
to help rewrite TRIG EQUATIONS

<u>Eg</u>: Definitions of tan, csc, sec, cot.

Pythagorean identities

Double $\angle$'s, etc.